Delta baryons in the separation geometry model.

Abstract

Extension of the separation geometry model of baryon structure from physics/0109024 and hep-ph/0201270 to the spin 3/2 Delta baryons. Theoretically derived masses in MeV; $\Delta^{++} = 1238.5$, $\Delta^{-} = 1243.4$, $\Delta^{0} = 1233.9$, $\Delta^{+} = 1232.6$ with the first one differing considerably from the quoted empirical value. Mass difference values are discussed.

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1 Motivation

In two previous papers [2] the background ideas and methodologies of separation geometry were described in some detail. The purpose of this paper is to outline a simple method of extension of the calculation technique to the spin 3/2 baryon resonances Δ^{-}, Δ^{0}, Δ^{+}, Δ^{++} as an extension of the proton and neutron mass calculations presented previously and to test the result against the recent calculation of Capstick et. al [1] for the mass differences of these objects.

2 Brief background concepts

Separation geometry approaches the issue of physical structure from quite a different perspective to standard QFT. Instead of superimposing fields satisfying local gauge invariance on a background four-dimensional space-time continuum, separation geometry works with models of particles as geometry based on explicitly *local gauge dependent* dimensional decomposition of the four-dimensional space-time continuum. This decomposition is well defined and is isomorphic to the cardinality structure of the real number continuum; i.e. it presupposes that space-time is a real continuum. The dimensional decomposition reduces fields to a local-gauge dependent form which is found to be suitable for the calculation of masses of fundamental fermions and the vector gauge bosons; in large measure because the problem of infinities associated with renormalisation of QFT's and running gauge couplings are eliminated in such a local gauge dependent approach; which fixes the coupling scale as a result of fixing the local-gauge (local phase) of the objets but in a way which allows for a natural transition to local-gauge independent geometry in the continuum limit and in which the geometry representation theory is independent of the actual local 'gauge' selected so that the calculated masses are likewise ultimately independent of local phase information. Separation geometry appears to be complementary to QFT; it has strengths in precisely the areas that QFT/standard model is weak (fermion masses, free parameters, no logical underpinning of the origin of fermion generation structure, raison d'etre for gauge structure etc.) but is weak in precisely the areas that QFT is strong (calculation of dynamical parameters; decay times, cross-sections etc).

The geometries that define local-gauge *dependent* dimensionally decomposed fields were called *affine geometries* in the previous papers and their properties were defined and studied. The geometric invariants of these objects are assumed to manifest as physical observables in the continuum limit; intrinsic spin, charge, mass etc. For each invariant there is a physical observable and every invariant that can be defined corresponds to an observable. The geometries define bounded spaces which in the limit of continuous geometry generators must, because of the geometrical construction of the theory, define compact group symmetries with the exception of the foundation geometry (which is a one-dimensional interval whose length is a gauge-dependent property) which evolves in the continuum limit to a non-compact symmetry associated with translations in space. It is an unproven supposition that these local-gauge-dependent features lead, with dimensional 'reconstitution', to local-gauge *independent*, i.e. physical, fields although significant data has been retrodicted (along with some precision predictions) which lend support to this supposition. Separation geometry is also 'covertly' supersymmetric at the level of the quark structure; which is to say that its' discrete symmetry contains the analogue of supersymmetry transformations between current quark operator representations (as distinct from physical states!) but we will not explore that issue here; see [2].

As should be expected for a theory of fundamental structure the theory has extreme economy; there are essentially only two affine geometries of interest. These are the affine cubic and tetrahedral geometries (and their associated sub-geometries). In the calculation process the cube is reduced to tetrahedral equivalent sub-groups so the geometry of the tetrahedon and its' associated sub-geometries, along with the geometry of the real number continuum, constitute the essential geometric elements of the theory. All the physical observable structure is abstracted from just the geometry, ultimately, of a tetrahedron in various incarnations.

3 Calculation algorithm.

A set of rules has been developed [2] which makes the calculation process for the mass of particles relatively simple. These rules have been derived from the discrete version of QCD which is a consequence of the embedding of tetrahedral affine symmetry into cubic affine symmetry. Discretised QCD has an explicitly gauge-dependent discrete symmetry in colour space but has many features that resemble standard QCD.

The rest-mass calculation of a hadron in separation geometry is handled in pieces. Each 'piece', with an appropriate non-perturbative radiative correction, of mass is then added to give a total mass. The pieces are;

1; Constituent quark mass. This is due to the energy-momentum of the current quarks and is represented as the matrix order (the cardinality or number of matrix elements in a set) which, in the discrete version of QCD, is represented as the number of tetrahedral-equivalent matrix units in a six-tetrahedral-component vector object called the 'particle vector'. This is an irreducible representation of the symmetry.

2; Gluon energy; found in a similar way by adding up the matrix order of the analogous representations of the gluons which couple to the particle vector.

3; Current quark intrinsic mass; this is also expressed in terms of tetrahedral units and represents the effective rest-mass of the individual quarks. This is calculated from matrix 'operators', also formed as irreducible six-tetrahedral-component objects, which couple to the particle vector to describe the state present.

4; Current quark separation energy; rather like a potential energy of separation of the current quarks due to the strong interaction at the energy scale of the calculation which is fixed by the symmetry. These are termed U(1) components in the text because there is the suggestion that they are related to a discrete U(1) symmetry. (The electromagnetic potential energy of separation of the current quarks is automatically incorporated in the the current-quark 'operators' structure and associated radiative correction - which are non-perturbative and governed by a semi-empirically determined ansatz; see below).

The details in the case of the nucleons are covered in the mentioned papers [2]. One identifies the the order of the various components and then multiples by the matrix order of the tetrahedral group(s) which is either 22 or 24 elements depending on whether the two group generators are acting as massless intrinsic spin generators (22 elements) or not (in which case you have 24 massive elements); and then one adds them all up. For second and third generation quarks, scalar components arising explicitly from the Higgs field must be added to the current quark masses calculated but these are not required for the first generation quarks (which do not acquire scalar components in the separation geometry model; at least not explicitly - analogous to treating the mass as dynamical in origin independent of the Higgs field).

All components, with the exception of 4, acquire a simple multiplicative radiative correction of the form $\mathcal{R} = (1 + \alpha_{q^2=m_e^2} + G_f)$ where α is the electro-magnetic coupling strength and m_e is $\approx$ the electron rest mass (which is roughly equivalent to the mass of a single tetrahedral unit) and G_f is the weak coupling constant expressed as a dimensionless number to represent its' effective strength with respect to α_{em} at the low energy scale; here of order 10^{-5}. Here the digit '1' in $\mathcal{R}$ is also functionally the strong coupling constant when applied to quarks - the scale of α_s is fixed by the tetrahedral symmetry at unity (this is the great advantage of calculating in an explicitly local-gauge dependent discrete environment where one does not have a running coupling to deal with but instead has a fixed point scale; all mass calculations reduce to the tetrahedral scale - roughly 0.5MeV - and the radiative correction is universal across fermion species as we have in the discrete scheme quark/lepton unification at the level of tetrahedral symmetry). In this sense then, component 4 has a multiplicative radiative correction of $\mathcal{R}^f = \alpha_s = 1$ when applied to strongly interacting particles.

After performing the appropriate summation and applying the non-perturbative radiative correction the mass of the particle can then be calculated by, for example (and this is usually the simplest way), taking the ratio with the electron rest mass which in separation geometry is defined by the order of the tetrahedral T_r group which has 4!=24 elements in its' matrix representation and two generators. The generators manifest as massless intrinsic spin generators in the transition to a field theory so that the remaining 22 T_r matrix elements, with radiative correction $\mathcal{R}$, defines the electron rest mass;

$$\mathcal{R}.(T_r \text{ (No. of irrep. matrix elements) } - T_r \text{ (generators)}) = \mathcal{R}.(4!-2) \equiv 0.5110000\ MeV \quad (1)$$

It is then a simple matter to convert any matrix order expression, M for the mass of a hadron into MeV;

$$\text{mass (MeV)} = \frac{M}{\mathcal{R}.22}.0.511$$

where M includes any radiative corrections as described. The multiplicative radiative correction $\mathcal{R}$ is a dimensionless number whose value is approximately 1.0073115 and represents the sum $(1 + \alpha^{-1}_{q^2=m_e^2} + G_f)$. Thus matrix order expressions have the dimension of energy.

4 Modifications to calculation algorithm for Δ baryons

The delta baryons Δ^+, Δ^{++}, Δ^0 and Δ^- are spin 3/2 fermions with three current quarks; $I(J^P) = \frac{3}{2}(\frac{3}{2}^+)$. For mass calculations of baryons containing only first generation quarks we have the following mass components to compute;

1. Constituent quark energy.
2. Current quark mass.
3. Gluon energy.
4. Current quark (strong or U(1)) potential terms.

We expect that a shift in spin state will essentially leave 2,3 and 4 unchanged in comparison with the proton and neutron calculations (modulo adjustments for the different current quark content in individual Δ's) but result in an increase in the value of item 1. The simplest ansatz that could be proposed is to increase the effective constituent energy by the equivalence of one unit of spin; that is two units of constituent quark energy (each unit representing one half-integer of spin). This is the same as multiplying the constituent energy of the nucleon baryon by a factor of 5/3. The actual quark content of the baryon is carried in the current quark representation - not in the constituent 'particle vector' representation which represents energy above and beyond the current quark rest mass due to current quark momentum. This procedure seems to work well for the delta masses.

5 The calculations

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We will compute the current quark masses for each of the four species first. The Δ^{++} consists of three up quarks and the current quark representation is;

$$\text{strong component} = \begin{pmatrix} \mathcal{I} & q^* & q^* \\ q^* & \mathcal{I} & q^* \\ q^* & q^* & \mathcal{I} \end{pmatrix}, \quad \text{E.M. component} = \begin{pmatrix} I & q^* & q^* \\ q^* & I & q^* \\ q^* & q^* & I \end{pmatrix} \tag{2}$$

and the matrix orders are read off the table; in the strong component each q^* and each $\mathcal{I}$ delivers 4! matrix elements and in the E.M. table each q^* gives a (4!-2) and each identity a 4! of elements. There are no cancellations. This gives 420 matrices. There is a parity doubling to 840.

To calculate the U(1) components for a baryon we use a triangle diagram; we place one of the current quarks at each vertex and each line of the triangle represents a potential energy of separtion. Each line between two quarks has an energy determined by the quarks at either end of the line. An up-up bond has (4!-2) matrix order, and u-d line has 2(4!-2) order and a d-d type line has matrix order 4.(4!-2). We sum over the triangle so the Δ^{++} has a U(1) matrix order of just 3.(4!-2). The Δ^{++} total current quark mass by the algorithm is then;

$\Delta^{++} = \mathcal{R}840 + 3(4! - 2)$. (Note that the U(1) component does not pick up a radiative correction).

For the Δ^{+} and Δ^{0} we have current quark masses identical the the proton and neutron respectively which have been calculated in hep-th/0109077 as;

$\Delta^{+} = \mathcal{R}564 + 4(4! - 2)$

and;

$\Delta^{0} = \mathcal{R}576 + 6(4! - 2)$

and finally for the Δ^{-} we have three down quarks as per the chart;

$$\text{strong component} = \begin{pmatrix} I & q^* & I \\ I & I & q^* \\ q^* & I & I \end{pmatrix}, \quad \text{E.M. component} = \begin{pmatrix} I & q & I \\ I & I & q \\ q & I & I \end{pmatrix} \tag{3}$$

which has the order 15.4! + 3.(4!-2). With parity doubling and the addition of the U(1) for three down-quarks = 12(4!-2) we obtain;

$\Delta^{-} = \mathcal{R}.852 + 12(4! - 2)$.

The glue order for the baryon is easily calculated as $\mathcal{R}(6.4!)^2$ (this is identical to the value for the nucleons) and the constituent quark energy as;

$$\mathcal{R}.\frac{5}{3}(6(4! - 2).6.4!)$$

(Notice the $\frac{5}{3}$ factor which is the boost to the constituent energy in the transition from the nucleon expression for the constituent mass to the Δ baryons). An easy calculation then gives the following masses;

Δ^{++} = 1238.51 MeV.
Δ^{+} = 1232.61 MeV
Δ^{0} = 1233.90 MeV
Δ^{-} = 1243.36 MeV.

Note that $\Delta^0 - \Delta^+ \approx 1.3MeV$ and $\Delta^- - \Delta^{++} \approx 4.9MeV$ so that $3(\Delta^0 - \Delta^+)\approx\Delta^- - \Delta^{++}$ broadly in agreement with model expectations given by Jenkins et. al [3] and Capstick et al[1] who predict a value of 1.5MeV for these mass differences. The calculated mass of the Δ^{++} in particular differs significantly from the standard quoted empirical value however;

$$\Delta^{++} = 1230.9\pm0.3, \quad \Delta^{+} = 1234.9\pm1.4, \quad \Delta^{0} = 1233.6\pm0.5.$$

Note that the relation [3];

$$\Delta_3 = \Delta^+ - \Delta^- - 3(\Delta^+ - \Delta^0) = \frac{\epsilon^{''}\epsilon^{'}}{\mathrm{N}_c^3} \approx 10^{-3} \tag{4}$$

quoted in [1] is violated with the derived masses in this study as we obtain (changing signs in accordance with the mass heirarchy);

$$\Delta_3 = \Delta^- - \Delta^{++} - 3(\Delta^0 - \Delta^+)\approx 1MeV.$$

Here two ϵ's are isopsin violating parameters for the strong and electromagnetic mass splitting respectively suggesting that in the model presented these isospin symmetries are broken. Most of the symmetry breaking appears to come from the strong interaction U(1) components. From current quark triangle diagrams one easily obtains;

$$\Delta^-_{U(1)} - \Delta^{++}_{U(1)} = 3.(\Delta^0_{U(1)} - \Delta^+_{U(1)}) + 1.52MeV$$

However, if we ignore the strong-interaction U(1) components completely separation geometry give an exact mass relation;

$$\Delta^- - \Delta^{++} = \Delta^0 - \Delta^+ \tag{5}$$

This relation arises because firstly the theory of current quarks in separation geometry does not distinguish, in mass calculations, the sign of an of an electromagnetic (++ or - etc) charge; only its' absolute magnitude. Secondly, the other factor that contributes to relation eq.(5), and distinguishes the theory from the standard model, is that it is supersymmetric between the scalar and spinor components of the current quark operators with both pieces carrying colour information; as the reader can check from the standard representation charts in [2]. It is possible eq.(5) is the supersymmetric analogue of eq.(4) and, in that circumstance, would then imply an exact preservation of electro-magnetic isospin symmetry in separation geometry. Note that relation eq.(5) does not represent the *physical* Δ states but states stripped of current-quark strong interaction potential energy terms.

References

[1] S Capstick, R Workman; Phys Rev. D59 (1999)014032 nucl-th/9807025 see also R Workman Phys. Rev. C56 (1997) 1645-1646 nucl-th/9705021

[2] G Filewood physics/0109024 and hep-ph/0201270

[3] E Jenkins R.F Lebed Phys. Rev. D52 282 (1995)